Chapter–6 Motion

Motion: An object is said to be in motion if it changes its position with respect to time.

Rest: An object is said to be at rest if it does not change its position with respect to time.

**** Two parameters required to specify the position of an object:**
1. The distance of the object from the chosen point (Origin)
2. The direction of the object from the chosen point (Origin)

Distance: The total length of the path followed by an object is called its distance.

Displacement: It is the shortest distance measured from initial point to final point of an object.

Note:
** Distance $\geq$ Displacement
** Displacement of an object can be zero when the final position of an object coincides with the initial point.

Uniform Motion: The motion of an object is said to be **uniform** if it covers equal distances in equal intervals of time.

Non-Uniform Motion: The motion of an object is said to be **non-uniform** if it covers unequal distances in equal intervals of time.

Speed: The distance covered by an object per unit time is called its **speed**.

$$v = \frac{\text{distance covered (s)}}{\text{time (t)}}$$

Velocity: Velocity is defined as the speed of an object in a particular direction.

SI Unit: The SI unit of speed and velocity is **m/s** (metre per second).

Average Speed:

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

Average Velocity:

$$\text{Average velocity} = \frac{u+v}{2}$$

where u is initial velocity and v is final velocity.

Acceleration (a): The rate of change of velocity of an object in motion is called **acceleration**. It is given by,

$$a = \frac{\text{change in velocity}}{\text{time}} = \frac{v - u}{t}$$

Deceleration (Retardation): The rate of change of velocity (decreasing) of an object is called **deceleration** or **retardation**.

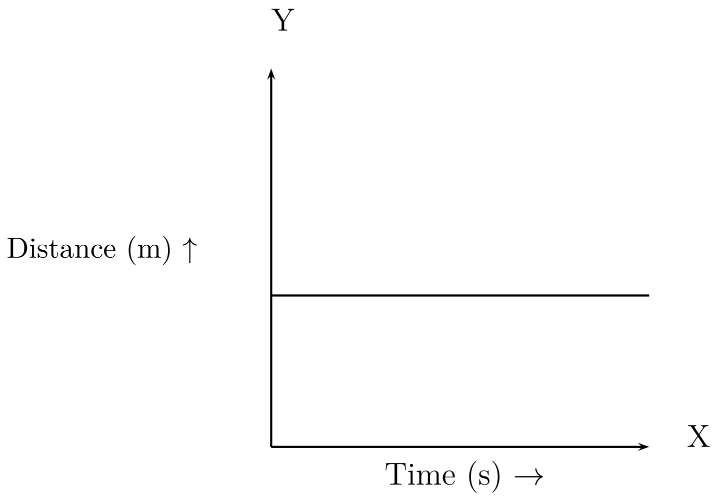
SI Unit of Acceleration: The SI unit of acceleration and deceleration is $\mathbf{m/s^2}$.

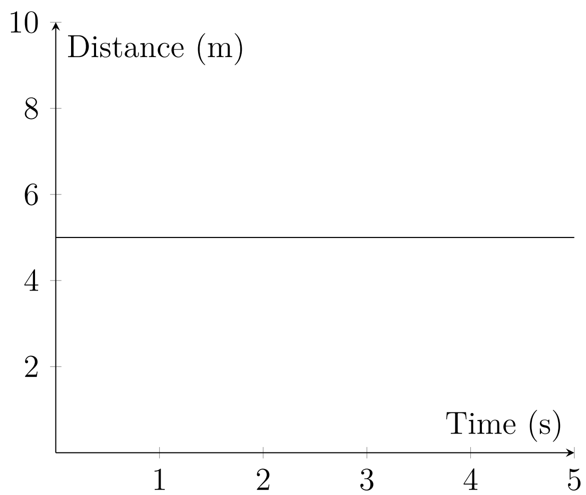
Uniform Acceleration: The acceleration of an object is said to be **uniform** if the velocity increases or decreases by equal amounts in equal intervals of time.

Non-Uniform Acceleration: The acceleration of an object is said to be **non-uniform** if the velocity changes by unequal amounts in equal intervals of time.

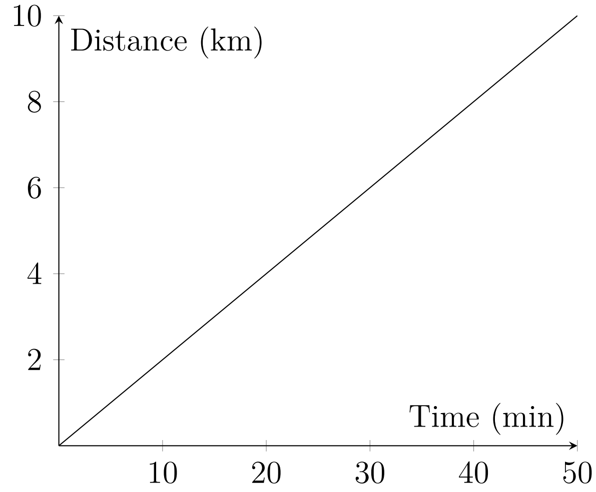
Graphical Representation of Motion

(a) Object at Rest: From the figure, we conclude that the distance-time graph of an object at rest is a straight line parallel to the time axis.



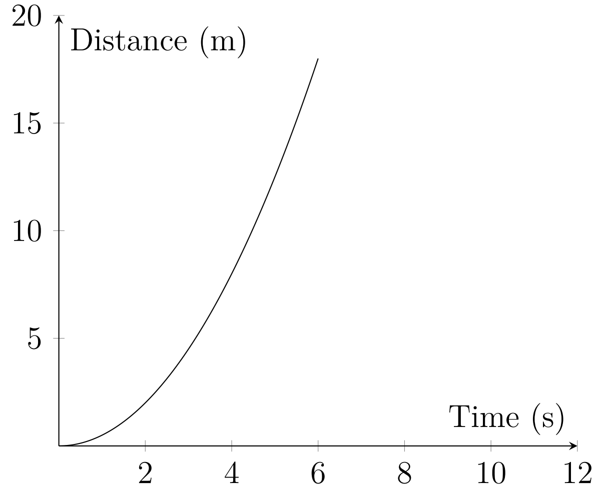


(b) Uniform Motion: From the figure, the distance-time graph of uniform motion is a straight line passing through the origin.



Non-Uniform Motion

From the figure, the distance-time graph of non-uniform motion is a curved line.

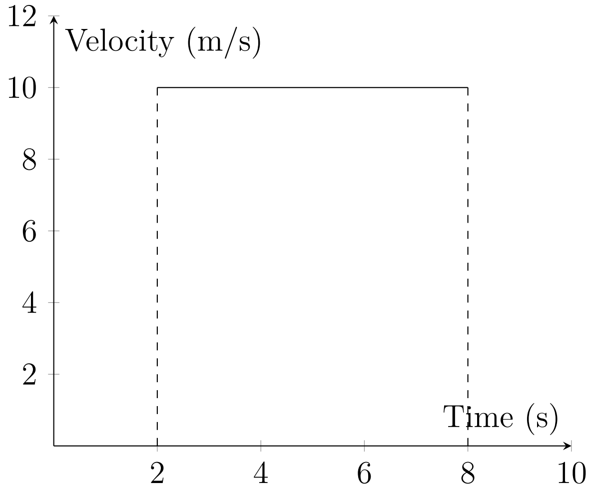


Distance from Velocity-Time Graph

To find the distance covered between time t_1 and t_2, we draw perpendiculars from points A and B.

Distance covered is equal to the area under the velocity-time graph.

$$\text{Distance} = \text{Area of rectangle} = \text{length} \times \text{breadth}$$



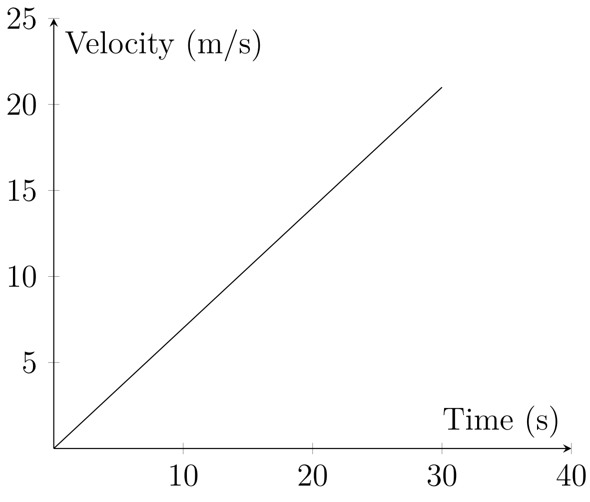
Uniformly Accelerated Motion

From the graph, distance travelled is equal to the area under the velocity-time graph.

$$\text{Distance} = \text{Area of trapezium}$$

$$= \text{Area of rectangle} + \text{Area of triangle}$$

$$= AB \times BC + \frac{1}{2} \times AD \times DE$$



Equations of motion:

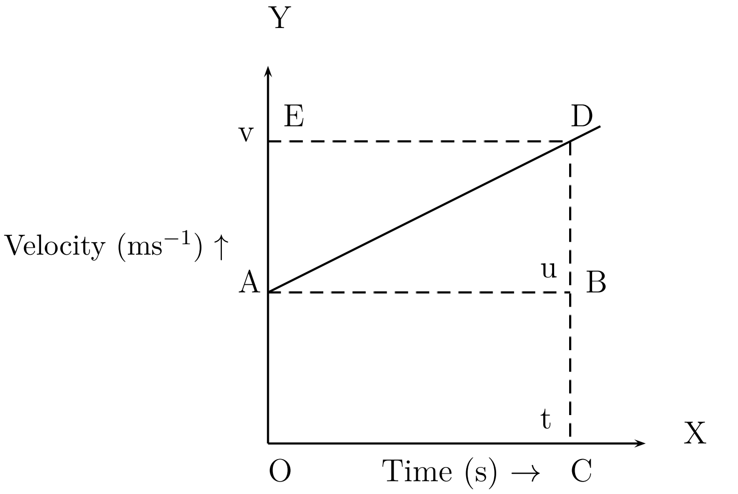
(i) $\mathbf{v = u + at}$

(ii) $\mathbf{s = ut + \frac{1}{2}at^2}$

(iii) $\mathbf{v^2 = u^2 + 2as}$

where, v = final velocity
u = initial velocity
t = time taken
a = acceleration
s = distance covered during time 't'

Equation for V-T relation of a motion, $\boldsymbol{v=u+at}$



Consider a V-T graph of an object moving under uniform acceleration from the initial velocity (u) to the final velocity (v) during time (t).

From the graph,

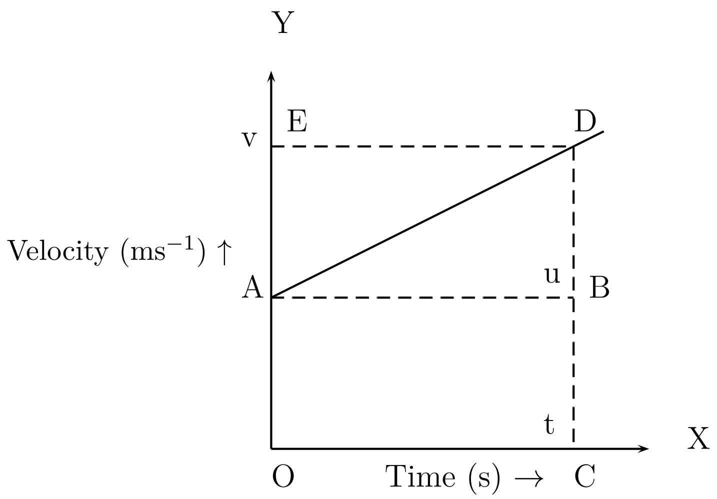
Let OA = BC = u
OE = CD = v
OC = AB = t

Then, CD = BD + BC
$\implies$ v = BD + u
$\implies$ BD = v - u ... (1)

Also, acceleration, a = $\dfrac{\text{Change in velocity}}{time}$
$\implies$ a = $\dfrac{BD}{t}$
$\implies$ BD = at ... (2)

Using equation (1) and (2),
at = v - u
$\implies$ **v = u + at**

Equation for position-time relation $s = ut + \dfrac{1}{2}at^2$



Consider an object has covered a distance 's' under uniform acceleration 'a' during time 't' . From the graph, this distance 's' is given by the area of trapezium under V-T graph AD.

Then,

distance, s = Area of trapezium OADC

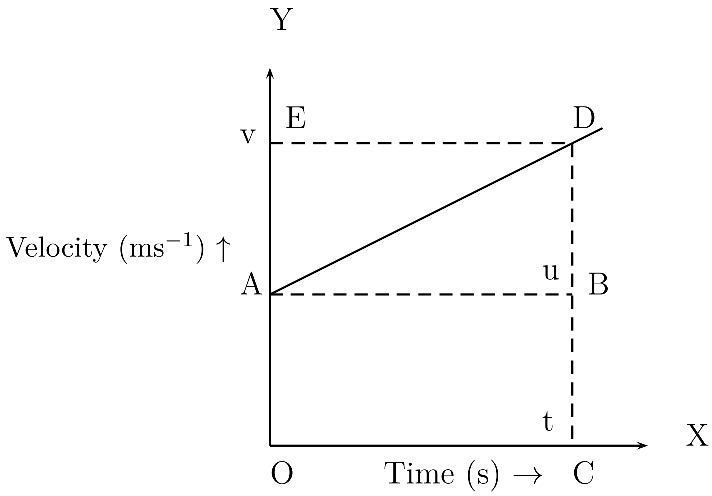
$\implies$ s = area of rectangle OABC + area of triangle ABD

$\implies$ s = OA × OC + $\frac{1}{2}$ AB × BD

$\implies$ s = u × t + $\frac{1}{2}$× t × at [$\because$ OA = u, AB = OC = t, BD = at]

$\implies$ $s = ut + \frac{1}{2}at^2$, which is the required expression.

Equation for position-velocity relation $v^2 = u^2 + 2as$



Consider an object has covered a distance 's' under uniform acceleration 'a' during time 't' . From the graph, this distance 's' is given by the area of trapezium under V-T graph AD.

Then,

distance, s = Area of trapezium OADC

$\implies$ s $= \dfrac{(OA + DC) \times OC}{2}$

$\implies$ s $= \dfrac{(u + v) \times t}{2}$

$\implies$ s $= \dfrac{(u + v)(u - v)}{2a}$ $\qquad [\because a = \dfrac{v-u}{t} \qquad \implies t = \dfrac{v-u}{a}]$

$\implies$ 2as $= v^2 - u^2$

$\implies$ $v^2 = u^2 + 2as$, which is the required expression.