Exercise : 12.6

1. Radii of the frustum are

$r_1 = 20\,cm$, $r_2 = 12\,cm$ and height $h = 6\,cm$.

Now, Slant height, $l = \sqrt{(r_1 - r_2)^2 + h^2} = \sqrt{(20-12)^2 + 6^2}$

$= \sqrt{8^2 + 6^2} = \sqrt{100} = 10\,cm.$

Then,

Vol. of the frustum $= \frac{1}{3}\pi h\left(r_1^2 + r_1 r_2 + r_2^2\right)$.

$= \frac{1}{\cancel{3}} \times 3.14 \times \cancel{6}^{2}\left(20^2 \times 20 \times 12 + 12^2\right)$

$= 3.14 \times 2 \; (400 + 240 + 144)$

$= 3.14 \times 2 \times 784$

$= 4923.52\,cm^3.$

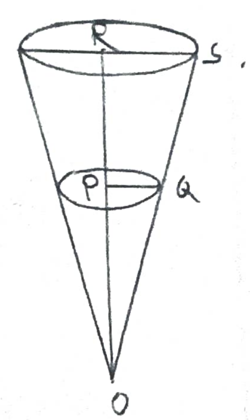
and curved surface area $= \pi l\,(r_1 + r_2)$

$= 3.14 \times 10\,(20+12)$

$= 3.14 \times 10 \times 32$

$= 1004.8\,cm^2.$

2. Here, height of the bucket, $h_1 = 16$ cm.
radius of the upper base, $r_1 = 18$ cm
radius of the lower base, $r_2 = 6$ cm



Then, 1) we have

$r_1 = 18$ cm, $r_2 = 6$ cm and $h_1 = 16$ cm.

considering the fig, we get

$PQ \parallel RS$

$\Rightarrow \frac{H}{h_2} = \frac{r_1}{r_2}$ $[\because H = OR, h_2 = OP]$

$\Rightarrow \frac{h_1 + h_2}{h_2} = \frac{r_1}{r_2}$ $\Rightarrow \frac{16 + h_2}{h_2} = \frac{18}{6}$

$\Rightarrow 16 + h_2 = 3h_2$ $\Rightarrow 2h_2 = 16$.

$\Rightarrow h_2 = 8$ cm

$\therefore$ height of the cone which the bucket is a part, $H = h_1 + h_2$

$= 16 + 8$

$= 24$ cm.

ii) capacity of the bucket $=$ Vol. of the bucket

$$= \frac{\pi h_1}{3}\left(r_1^2 + r_2^2 + r_1 r_2\right)$$

$$= \frac{22 \times 16}{7 \times 3}\left(18^2 + 6^2 + 18 \times 6\right)$$

$$= \frac{22 \times 16}{21} \times 468$$

$$= 7844.57 \text{ cm}^3.$$

iii) Slant height of the bucket, $l = \sqrt{(r_1 - r_2)^2 + h_1^2} = \sqrt{18^2 + (18-6)^2}$

$$= \sqrt{400} = 20 \text{ cm}.$$

iv) surface area of the bucket $= \pi l (r_1 + r_2) + \pi r_2^2$

$$= \pi \left[l (r_1 + r_2) + r_2^2\right]$$

$$= \frac{22}{7}\left[20 (18 + 6) + 6^2\right]$$

$$= \frac{22}{7} \times \left[20 \times 24 + 36\right]$$

$$= \frac{22}{7} \times 516 = 1621.71 \text{ cm}^2.$$

3. Here, we get,

height of the frustum = 21 cm.

$r_1 = 35$ cm and $r_2 = 28$ cm

Now, capacity of the bucket. = vol. of the frustum

$$= \frac{\pi h}{3}(r_1^2 + r_2^2 + r_1 r_2)$$

$$= \frac{22 \times \cancel{21}}{\cancel{21}}(35^2 + 28^2 + 35 \times 32)$$

$$= 22 \times 2989 = 65758 \text{ cm}^3.$$

$$= \frac{65758}{1000} \ l = 65.76 \ l$$

4. Radius of the larger face of frustum, $r_1 = \frac{66 \times 7}{2 \times 22} = \frac{21}{2}$ cm.

and radius of the smaller face of frustum, $r_2 = \frac{44 \times 7}{22 \times 2} = 7$ cm.

height of the frustum = 12.6 cm.

$\therefore$ Volume of the frustum $= \frac{\pi h}{3}(r_1^2 + r_2^2 + r_1 r_2)$.

$$= \frac{22 \times \cancel{12.6}^{1.8}}{\cancel{7} \times 3} \left(\frac{441}{4} + 49 + \frac{21}{2} \times 7\right)$$

$$= \frac{22 \times 1.8}{3} \times \left(\frac{441 + 196 + 294}{4}\right)$$

$$= \frac{22 \times 1.8}{3} \times \frac{931}{4} = \frac{36867.6}{12}$$

$$= 3072.3 \text{ cm}^3$$

5. Here, $r_1 = \frac{7}{2} = 3.5$ cm.

$r_2 = \frac{4.2}{2} = 2.1$ cm.

$h = 12$ cm.

Now, capacity of the glass tumbler = vol. of the frustum

$$= \frac{\pi h}{3}\left(r_1^2 + r_2^2 + r_1 r_2\right)$$

$$= \frac{3.14 \times \cancel{12}^{4}}{\cancel{3}} \left[(3.5)^2 + (2.1)^2 + 3.5 \times 2.1\right]$$

$$= 12.56 \left[12.25 + 4.41 + 7.35\right]$$

$$= 12.56 \times 24.01 = 301.57 \text{ cm}^3.$$

6. Here, $r_1 = \frac{88}{2} = 44$ cm $\quad r_2 = \frac{66}{2} = 33$ cm and $l = 21$ cm.

$\therefore$ total surface area of the frustrum

$= \pi\left[l(r_1+r_2) + r_1^2 + r_2^2\right]$

$= \frac{22}{7}\left[21(44+33) + 44^2 + 33^2\right]$

$= \frac{22}{7}(1617 + 1936 + 1089) = \frac{22}{7} \times 4642$

$= 14589.14$ cm^2.

7. Here, $r_1 = 17$ cm $\quad r_2 = 8$ cm and $h = 12$ cm

Now, volume of frustum $= \frac{\pi h}{3}(r_1^2 + r_2^2 + r_1 r_2)$

$= \frac{3.14 \times \cancel{12}^{4}}{\cancel{3}}\left[17^2 + 8^2 + 17\times 8\right]$

$= 12.56\ (289 + 64 + 136)$

$= 12.56 \times 489 = 6141.84$ cm^3

$= 6.14\ l$.

$\therefore$ capacity of milk $= 6.14\ l$.

$\therefore$ cost of milk $= ₹\ 20 \times 6.14$

$= ₹\ 122.8$.

Then, curved surface area of the container

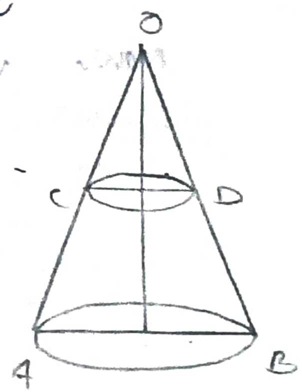
$= \pi l (r_1 + r_2)$

$= 3.14 \times \sqrt{12^2 + 9^2} \times 25$.

$= 3.14 \times 15 \times 25$.

$= 1177.5\ cm^2$.

8. As a plane cut offs at the middle of the height & We have R and R/2 as the radii of the lower base and upperbase of the frustum



Now, Vol. of the conical part

$=$ Vol. of the portion of CDO.

$= \frac{1}{3}\pi\left(\frac{R}{2}\right)^2 \times \frac{h}{2} = \frac{\pi h R^2}{6 \times 4}$.

and vol. of the frustum ABCD

$= \frac{\pi h}{3 \times 2}\left[R^2 + \left(\frac{R}{2}\right)^2 + \frac{R}{2} \times R\right]$

$$= \frac{\pi h}{6}\left[R^2 + \frac{R^2}{4} + \frac{R^2}{2}\right] = \frac{\pi h R^2}{6}\left[1 + \frac{1}{4} + \frac{1}{2}\right]$$

$$= \frac{\pi h R^2}{6} \times \frac{7}{4}.$$

$\therefore$ the reqd. ratio $= \dfrac{\text{Vol. of conical part}}{\text{Vol. of the frustrum}} = \dfrac{\frac{\pi R^2 h}{24}}{\frac{7\pi R^2 h}{24}}$

$$= \frac{1}{7} = 1:7.$$

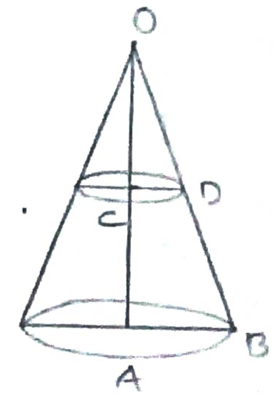
9. Let r_1 and r_2 be the respective radius with respect to small cone and given cone.

We have, $V_1 : V_2 = 8:19$.

$$\Rightarrow \frac{V_1}{V_2} = \frac{8}{19} \quad \Rightarrow \frac{V_2}{V_1} = \frac{19}{8}$$

$$\Rightarrow \frac{V_2+V_1}{V_1} = \frac{19+8}{8} = \frac{27}{8} \quad \text{(componendo)}.$$

$$\Rightarrow \frac{V}{V_1} = \frac{27}{8} \quad \text{— (1)}, \text{ where } V = V_1 + V_2.$$



Let $OC = h_1$ and $AC = h_2$

Then from (i) we get

$$\frac{\frac{1}{3}\pi h_2^2 (h_1 + h_2)}{\frac{1}{3}\pi h_1^2 h_1} = \frac{27}{8}$$

$$\Rightarrow \frac{h_2^2}{h_1^2}\left(\frac{h_1+h_2}{h_1}\right) = \frac{27}{8} \quad \text{— (ii)}$$

But $\frac{AB}{CD} = \frac{AO}{CO}$

$$\Rightarrow \frac{h_2}{h_1} = \frac{h_1+h_2}{h_1} \quad \text{— (iii)}$$

using (iii) in (ii) we get

$$\frac{h_2^2}{h_1^2} \times \frac{h_2}{h_1} = \frac{27}{8} \Rightarrow \frac{h_2^3}{h_1^3} = \frac{27}{8}.$$

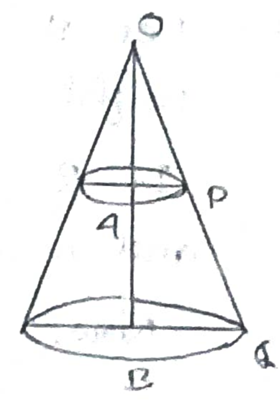
$$\Rightarrow \frac{h_2}{h_1} = \frac{3}{2}. \Rightarrow \frac{h_1}{h_2} = \frac{2}{3}.$$

$\therefore$ the reqd. ratio $= 2:3$.

10. Here, $AO = h_1$ $\quad AB = h_2$.

$\therefore h_1 + h_2 = 25$ cm. $= OB$

and $AP = r_1$ and $BQ = r_2$.



Then,

$\pi r_1^2 = 154$.

$\Rightarrow r_1^2 = \frac{154 \times 7}{22} = 49$.

$\therefore r_1 = 7$ cm.

Then as $\triangle OAP \sim \triangle OBQ$, we get

$\frac{OA}{OB} = \frac{AP}{BQ}$.

$\Rightarrow \frac{25 - h_2}{h_1 + h_2} = \frac{7}{10}$. $\quad \Rightarrow \frac{25 - h_2}{25} = \frac{7}{10}$.

$\Rightarrow 250 - 10h_2 = 25 \times 7$.

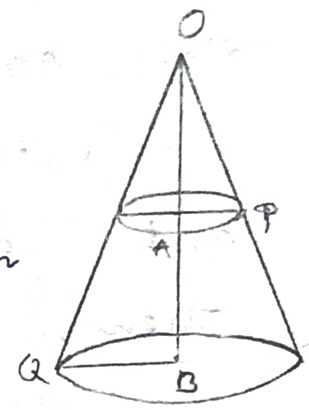
$\Rightarrow 10h_2 = 250 - 175 = 75$.

$\Rightarrow h_2 = 7.5$ cm.

$\therefore$ the reqd. height is 7.5 cm.

11. Let H be the height of the cone and h be the height of the frustum so formed.

If R be the radius of the lower base of the frustum and r be the radius of the upper base of the frustum



Now, by question, we get

$$\frac{\pi l' (R+r)}{\pi R l} = \frac{15}{16}$$

$$\Rightarrow \frac{l'}{l}\left(\frac{R+r}{R}\right) = \frac{15}{16} \qquad \Rightarrow \frac{l'}{l}\left(1+\frac{r}{R}\right) = \frac{15}{16}$$

$$\Rightarrow \frac{l'}{l}\left(1+\frac{l-l'}{l}\right) = \frac{15}{16} \qquad \left[\because \Delta OAP \sim \Delta OBQ, \text{ so } \frac{r}{R} = \frac{l-l'}{l}\right]$$

$$\Rightarrow \frac{l'}{l}\left(2-\frac{l'}{l}\right) = \frac{15}{16} \qquad \Rightarrow \frac{l'^2}{l^2} - \frac{2l'}{l} + \frac{15}{16} = 0.$$

$$\Rightarrow 16\left(\frac{l'}{l}\right)^2 - 32\left(\frac{l'}{l}\right) + 15 = 0.$$

$$\Rightarrow \frac{l'}{l} = \frac{32 \pm \sqrt{32^2 - 4\times16\times16}}{32} = 1 \pm \frac{\sqrt{32\times2}}{32}.$$

$$\Rightarrow \frac{l'}{l} = 1 \pm \frac{8}{32} = \frac{90}{32} \text{ or } \frac{24}{32}.$$

$$\Rightarrow \frac{l'}{l} = \frac{24}{32} \quad \left[\because \frac{90}{32} \text{ is neglected as } l' < l\right]$$

$$\Rightarrow \frac{l'}{l} = \frac{3}{4}.$$

Again as $\triangle OAP \sim \triangle OBQ$ we get

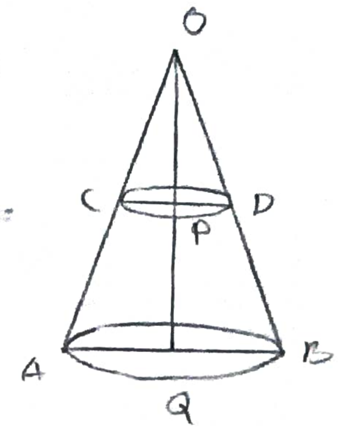
$$\Rightarrow \frac{l'}{l} = \frac{h}{H} = \frac{3}{4}$$

Hence $h = \frac{3}{4} H$ Proved.

12. Here, $OQ = 24$ cm.

Let $QB = R$ and $PD = r$ and $PQ = h$.

It clears that $\triangle OPD \sim \triangle OQB$.



So, $\frac{OP}{OQ} = \frac{PD}{QB} \Rightarrow \frac{OQ - PQ}{OQ} = \frac{r}{R}.$

$$\Rightarrow \frac{24-h}{24} = \frac{r}{R} \Rightarrow 1 - \frac{h}{24} = \frac{r}{R}.$$

$$\Rightarrow \frac{h}{24} = 1 - \frac{r}{R} \quad \text{——(1)}$$

Taking V be the vol. of the given cone & V' be the vol. of the frustrum

Then, $\frac{V'}{V} = \frac{19}{27}$.

$$\Rightarrow \frac{\frac{\pi h}{3}\left[R^2 + r^2 + Rr\right]}{\frac{\pi}{3} \times R^2 \times 24} = \frac{19}{27}.$$

$$\Rightarrow \frac{h\left(R^2 + r^2 + Rr\right)}{24R^2} = \frac{19}{27}.$$

$$\Rightarrow \frac{h}{24}\left[1 + \left(\frac{r}{R}\right)^2 + \frac{r}{R}\right] = \frac{19}{27}.$$

$$\Rightarrow \left(1 - \frac{r}{R}\right)\left[1 + \frac{r}{R} + \left(\frac{r}{R}\right)^2\right] = \frac{19}{27} \quad \left[\text{using (1)}\right]$$

$$\Rightarrow 1 - \left(\frac{r}{R}\right)^3 = \frac{19}{27} \quad \Rightarrow 1 - \frac{19}{27} = \left(\frac{r}{R}\right)^3.$$

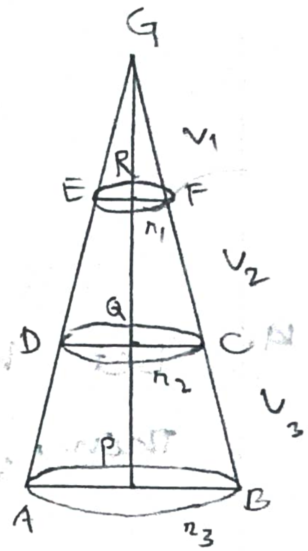
$\Rightarrow \frac{8}{27} = \left(\frac{r}{R}\right)^3 \quad \Rightarrow \frac{r}{R} = \frac{2}{3}$.

In ①, we get

$\frac{h}{24} = 1 - \frac{2}{3} = \frac{1}{3}$.

$\Rightarrow h = 8$ cm.

13. In the adjoining fig.



the cone is trisected such that

$GR = RQ = QP = h$.

$ER = r, \quad DQ = 2r \quad AP = 3r$.

Now, $\frac{V_1}{V_2} = \frac{\text{Vol. of top cone}}{\text{Vol. of frustum just after cone}}$

$= \frac{\frac{1}{3}\pi r^2 h}{\frac{1}{3}\pi h\left[(2r)^2 + 2r \times r + r^2\right]}$

$= \frac{r^2}{4r^2 + 2r^2 + r^2} = \frac{r^2}{7r^2} = \frac{1}{7}$.

$\therefore \; V_1 : V_2 = 1:7 \quad \text{--- (i)}$

Now, $\dfrac{V_2}{V_3} = \dfrac{\frac{1}{3}\pi h\left[(2r)^2 + 2r \times r + r^2\right]}{\frac{1}{3}\pi h\left[(3r)^2 + 3r \times 2r + (2r)^2\right]}$

$= \dfrac{4r^2 + 2r^2 + r^2}{9r^2 + 6r^2 + 4r^2} = \dfrac{7r^2}{19r^2} = \dfrac{7}{19}$.

$\therefore \; V_2 : V_3 = 7:19 \quad \text{--- (ii)}$.

Combining (i) and (ii) we get

$V_1 : V_2 : V_3 = 1:7:19$.

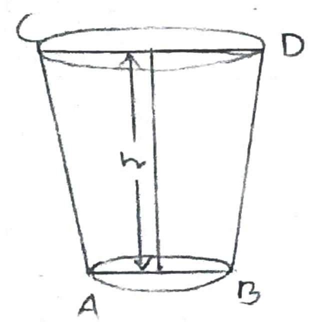
$\pi r^2 h$

14. Let h be the height of the bucket

Then, Vol. of bucket $= \dfrac{\pi h}{3}\left(28^2 + 7^2 + 28 \times 7\right)$

$\Rightarrow \; 45584 = \dfrac{22h}{3 \times 7}\left(784 + 49 + 196\right)$

$\Rightarrow \; 45584 = \dfrac{22h}{3 \times 7} \times 1029 = 12 \cdot 29$



$\Rightarrow h = \dfrac{45584 \times 21}{22 \times 1029} = 42.29$ cm.

Now, surface area $= \pi l (R+r) + \pi r^2$

$= \dfrac{22}{7} \left\{ \sqrt{h^2 + (R-r)^2}\,(R+r) + r^2 \right\}$

$= \dfrac{22}{7} \left\{ \sqrt{(42.29)^2 + 21^2} + 35^2 + 7^2 \right\}$

$= \dfrac{22}{7} (47.22 + 35 + 49)$

$= \dfrac{22}{7} \times 131.22$.

$= 412.40$ cm^2.

$\frac{1}{2} \times 9 \times 12$

$\frac{1}{2}$ BP $\times$

27/6