Exercise : 12.5.

1. Let 'a' be the edge of the cube.

Then, surface area of the cube $= 6a^2$.

surface area of the cuboid $= 2\left(a \times \frac{1}{2}a + a \times a + \frac{1}{2} \times a \times a\right)$

$= 2a^2\left(\frac{1}{2} + 1 + \frac{1}{2}\right)$

$= 4a^2$.

$\therefore$ Reqd. ratio $= \frac{6a^2}{4a^2} = \frac{3}{2} = 3:2$.

2. Here, surface area of sphere = curved surface area of cone.

If 'r' be the radius of the sphere.

Then,

$4\cancel{\pi}r^2 = \cancel{\pi}r'l$, where r' = radius of cone = 12 cm.

and $l = \sqrt{r'^2 + h^2} = \sqrt{12^2+16^2} = 20$ cm.

$\Rightarrow 4r^2 = 12 \times 20$

$\Rightarrow r^2 = 12 \times 5 = 60$.

$\therefore r = \sqrt{60} = 2\sqrt{15}$ cm.

Thus, radius of the sphere is $2\sqrt{15}$ cm.

3. Here,

Vol. of sphere = Vol. of cone.

$\Rightarrow \frac{4}{3}\cancel{\pi}R^3 = \frac{1}{3}\pi r^2 h.$

$\Rightarrow 4R^3 = 6 \times 6 \times 27.$

$\Rightarrow R^3 = \frac{\overset{3}{\cancel{6}} \times \overset{3}{\cancel{6}} \times 27}{\cancel{4}}$

$\therefore R = 9$ cm.

Now, surface area of the sphere

$= 4\pi r^2 = 4 \times \frac{22}{7} \times 9 \times 9$

$= 1018.29$ cm^2.

4. Here, diameter of the sphere = 28 cm.

$\therefore$ radius = 14 cm.

and diameter of cone = 7 cm.

$\therefore$ radius $= \frac{7}{2}$ cm.

height of the cone = 4 cm.

Let 'n' be the no. of cone.

Then, vol. of sphere = n × vol. of cone.

$$\Rightarrow \frac{4}{\cancel{3}} \times \cancel{\pi} \times 14^3 = n \times \frac{1}{\cancel{3}} \times \cancel{\pi} \times \left(\frac{7}{2}\right)^2 \times 4.$$

$$\Rightarrow 4 \times 14^3 = n \times \frac{49}{\cancel{4}} \times \cancel{4}$$

$$\therefore n = \frac{4 \times 14 \times 14 \times 14}{49} = 224.$$

∴ The no. of cone is 224.

5. Here,

vol. of the hollow sphere = vol. of cylinder

$$\Rightarrow \frac{4}{3} \cancel{\pi} (R^3 - r^3) = \cancel{\pi} r'^2 h. \quad (r' = \text{radius of cylinder})$$

$$\Rightarrow \frac{4}{3} (5^3 - 3^3) = r'^2 \times \frac{8}{3}.$$

$$\Rightarrow \frac{4}{3} \times 98 = r'^2 \times \frac{8}{3}.$$

$$\Rightarrow r'^2 = \frac{\overset{49}{\cancel{98}} \times \cancel{3} \times \cancel{4}}{\cancel{8} \times \cancel{3}} = 49.$$

$\therefore r' = 7$ cm

Now, surface area of cylinder

$= 2\pi r h$

$= 2 \times \frac{22}{\cancel{7}} \times \cancel{7} \times \frac{8}{3} = \cancel{117}\ \frac{352}{3} = 117\frac{1}{3}\ cm^2.$

6\. Let $h = 28$ cm, $r = 12$ cm

$h' = 9\frac{1}{3} = \frac{28}{3}$ cm.

Let R be the radius of cylinder. Then,

Vol. of cone = volume of cylinder

$\Rightarrow \frac{1}{3}\cancel{\pi} r^2 h = \cancel{\pi} R^2 h'$

$\Rightarrow \frac{1}{\cancel{3}} \times 12^2 \times \cancel{28} = R^2 \times \frac{\cancel{28}}{\cancel{3}}$

$\Rightarrow R^2 = 12^2$

$\therefore R = 12$ cm.

Now, curved surface area of the cylinder

$= 2\pi R h = 2 \times \frac{22}{\cancel{7}} \times \overset{4}{\cancel{12}} \times \frac{\overset{4}{\cancel{28}}}{\cancel{3}} = 704\ cm^2.$

7. Radius of the cylinder = 7 cm.

Height of the cylinder = $20 \times 1.5 = 30$ cm.

Then, curved surface area of the cylinder

$= 2\pi rh = 2 \times \frac{22}{\cancel{7}} \times \cancel{7} \times 30$

$= 1320 \text{ cm}^2$.

and volume of the cylinder $= \pi r^2 h$

$= \frac{22}{\cancel{7}} \times 7 \times \cancel{7} \times 30$

$= 4620 \text{ cm}^3$.

8. We have,

diameter of the sphere = length of one side of cube = 4.2 cm.

$\therefore$ radius of the possible largest sphere = 2.1 cm.

Then, Volume of the largest sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{\cancel{7}} \times \overset{0.3}{\cancel{2.1}} \times (2.1)^2$

$= 38.808 \text{ cm}^3$

$= 38.81 \text{ cm}^3$.

Now, $\dfrac{\text{Volume of cube}}{\text{Volume of sphere}} = \dfrac{(4.2)^3}{38.81}$

$=$

9. Side of the cube = 16.8 cm.

Radius of the cone = $\dfrac{16.8}{2}$ = 8.4 cm.

Height of the cone = 16.8 cm.

Now, volume of the largest cone $= \frac{1}{3}\pi r^2 h$.

$= \frac{1}{\cancel{3}} \times \frac{22}{\cancel{7}} \times \overset{1.2}{\cancel{8.4}} \times \overset{2.8}{\cancel{8.4}} \times 16.8$

$= 1241.86 \text{ cm}^3$.

10. Here, diameter of the cone = 14 cm.

height of the cone = 8 cm

Then, volume of the cone $= \frac{1}{3} \times \frac{22}{\cancel{7}} \times \cancel{7} \times 7 \times 8$

$= 410.67 \text{ cm}^3$.

Let r be the radius of the inner sphere

Then, Volume of the hollow sphere $= \frac{4}{3}\pi(R^3 - r^3)$

$$= \frac{4}{3}\pi(5^3 - r^3) \quad \left[\begin{array}{l} \because \text{external diameter} = 10 \text{ cm} \\ \therefore \text{radius} = 5 \text{ cm} \end{array}\right].$$

Taking volume of hollow sphere = volume of cone.

$$\Rightarrow \frac{4}{\cancel{3}} \times \frac{22}{\cancel{7}}(5^3 - r^3) = \frac{1}{\cancel{3}} \times \frac{22}{\cancel{7}} \times 7 \times 7 \times 8$$

$$\Rightarrow \cancel{4}(5^3 - r^3) = 49 \times 2 \times \cancel{4}$$

$$\Rightarrow 125 - r^3 = 98.$$

$$\Rightarrow r^3 = 125 - 98 = 27.$$

$$\therefore r = 3 \text{ cm}.$$

Hence, the internal diameter is 6 cm.

11. Here, radius of hemisphere = 7 cm.

radius of the cone = 3.5 cm.

Then, volume of the hemisphere $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{\cancel{7}} \times 7 \times 7 \times \cancel{7}$

$= 718.67 \text{ cm}^3$.

and volume of cone $= \frac{1}{3}\pi r^2 h$.

$= \frac{1}{3} \times \frac{22}{7} \times (3.5)^2 \times h$.

$= \frac{269.5}{21} h$.

Taking vol. of hemisphere = vol. of cone

$\Rightarrow 718.67 = \frac{269.5}{21} h$.

$\Rightarrow h = \frac{718.67 \times 21}{269.5} = 56$.

$\therefore$ height of the cone $= 56$ cm.

12. Here, we have,

curved surface area of cylinder = area of rectangle

$= 66 \times 28 \text{ cm}^2$

$= 1848 \text{ cm}^2$.

Now, curved surface area of cylinder $= 66 \times 28 \text{ cm}^2$.

$\Rightarrow 2\pi r h = 66 \times 28$

$\Rightarrow 2 \times \frac{22}{\cancel{7}} \times r \times \cancel{28}^{4} = 66 \times 28$.

$\Rightarrow r = \frac{\cancel{66}^{3} \times \cancel{28}^{14}}{\cancel{2} \times \cancel{22} \times 4} = 10.5$ cm.

$\therefore$ radius of the cylinder $= 10.5$ cm.

Then, volume of the cylinder $= \pi r^2 h$

$= \frac{22}{\cancel{7}} \times (10.5)^2 \times \cancel{28}^{4}$

$= 9702$ cm^3.

13. We have,

radius of the sector = 6 cm.

internal angle between the two radii $= 120°$.

Then, area of sector $= \frac{\pi \times \cancel{36}^{12} \times \cancel{120}}{\cancel{360}_{3}} = 12\pi$.

As, the sector rolled up and formed a cone

area of sector = lateral surface area of the cone.

$12\pi = \pi r \times 6$ [$\because$ radius of sector = slant height of cone]

$\therefore r = 2$ cm.

If h be the height of cone, we have

$h = \sqrt{l^2 - r^2} = \sqrt{6^2 - 2^2} = \sqrt{32} = 4\sqrt{2}$ cm.

Now, volume of the cone $= \frac{1}{3}\pi r^2 h$.

$= \frac{1}{3} \times \frac{22}{7} \times 2 \times 2 \times 4\sqrt{2}$

$= 23.70$ cm^3 (approx).

14. Here, we have,

Vol. of water in the cylindrical flask = Vol. of water in the conical flask.

$\Rightarrow \pi r^2 h = \frac{1}{3}\pi r^2 h$.

$\Rightarrow \left(\frac{2x}{3}\right)^2 \times h = \frac{1}{3} \times x^2 \times 2x$.

$\Rightarrow \frac{4x^2}{9} \times h = \frac{1}{3} \times x^2 \times 2x$.

$\Rightarrow h = \frac{\cancel{9}^{3} \times \cancel{x^2} \times \cancel{2}x}{2 \cancel{4} \cancel{x^2} \times \cancel{3}} = \frac{3x}{2}$.

$\therefore$ height of water in the cylindrical flask is $\frac{3x}{2}$ unit.

15. Let x_1 and x_2 be the radii of the first and second cone respectively

Then, vol. of cylinder = vol. of the two cones.

$\Rightarrow \pi r^2 h = \frac{1}{3}\pi x_1^2 h + \frac{1}{3}\pi x_2^2 h.$

$\Rightarrow \cancel{\pi} r^2 \cancel{h} = \frac{1}{3}\cancel{\pi}\cancel{h}(x_1^2 + x_2^2).$

$\Rightarrow r^2 = \frac{1}{3}(x_1^2 + x_2^2).$

$\therefore r = \sqrt{\frac{1}{3}(x_1^2 + x_2^2)}$ proved.

16. We have,

height of the cylinder = height of the cone $= h$.

radius of the cylinder = radius of the cone $= r$.

radius of the new cylinder $= \frac{r}{2}$.

Then, vol. of (cylinder + cone) = vol. of the new cylinder.

$\Rightarrow \pi r^2 h + \frac{1}{3}\pi r^2 h = \pi\left(\frac{r}{2}\right)^2 H$, where H = height of the new cylinder.

$\Rightarrow \pi r^2 h\left(1+\frac{1}{3}\right) = \pi r^2 \times \frac{H}{4}$

$\Rightarrow h\left(1+\frac{1}{3}\right) = \frac{H}{4}$.

$\Rightarrow \frac{4}{3}h = \frac{H}{4}$.

$\therefore H = \frac{16}{3}h$. Proved.

17. Here, vol. of first cuboid $= 15 \times 8 \times 5$ cm³.

vol. of second cuboid $= 20 \times 5 \times 4$ cm³.

vol. of cube $= 5^3$.

Let 'n' be the no. of cube whose side is 5 cm.

By question, $n \times 5^3 = 15 \times 8 \times 5 + 20 \times 5 \times 4$.

$\Rightarrow n = \frac{15 \times 8 \times 5 + 20 \times 5 \times 4}{5 \times 5 \times 5} = \frac{600+400}{125}$

$= \frac{1,000}{125} = 8.$

18. Diameter of the sphere = 6 cm.

$\therefore$ Radius, r = 3 cm.

Now, Volume of the sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 3^3 = \frac{22 \times 36}{7}$ cm^3.

Then, diameter of vessel = 12 cm.

$\therefore$ Radius, r' = 6 cm.

Let h be the increasing height in water level. Then

$\pi r'^2 h = \frac{4}{3}\pi r^3$.

$\Rightarrow \frac{22}{7} \times 36 \times h = \frac{22}{7} \times 36$.

$\therefore$ h = 1 cm.

Hence, 1 cm is raised in the water level.

19. Here, radius of cylinder = 14 cm

height of cylinder = 21 cm

and radius of sphere = 3.5 cm.

Let n be the no. of spheres so formed

Then, vol. of cylinder = $n \times$ vol. of sphere

$\Rightarrow \cancel{\pi} \times 14^2 \times 21 = n \times \frac{4}{3} \times \cancel{\pi} \times (3.5)^3.$

$\Rightarrow n = \frac{14 \times 14 \times 21 \times 3}{(3.5)^3 \times 4} = \frac{12348}{171.52}$

$= 71.99 = 72.$

20. Here, external diameter of the spherical shell = 16 cm.

$\therefore$ radius = 8 cm.

diameter of cylinder = 16 cm

$\therefore$ radius = 8 cm.

height of the cylinder = $9\frac{1}{3} = \frac{28}{3}$ cm.

Let r be the internal radius of the shell

Then, vol. of spherical shell = vol. of cylinder

$\Rightarrow \frac{4}{\cancel{3}} \pi (8^3 - r^3) = \pi \times 8^2 \times \frac{28}{\cancel{3}}$

$\Rightarrow \quad 4r^3 = 4 \times 8^3 - 8^2 \times 28 = 2048 - 1792 = 256.$

$\Rightarrow \quad r^3 = 64$

$\therefore \quad r = 4$ cm.

Thus, the reqd. internal diameter is 8 cm

21. Height of the cone = 8 cm.

radius of the cone = 6 cm.

Then, volume of the cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi \times 6^2 \times 8$ cm^3.

Now, radius of sphere = 0.2 cm. $= \frac{1}{5}$ cm.

$\therefore$ Volume of lead shot $= \frac{4}{3}\pi\left(\frac{1}{5}\right)^3 = \frac{4\pi}{3 \times 5^3}$.

Then, Total vol. of lead shots $= \frac{1}{6} \times$ vol. of cone.

Let n be the no. of lead shots

Then $n \times \frac{4\cancel{\pi}}{\cancel{3} \times 5^3} = \frac{1}{6} \times \frac{1}{\cancel{3}} \times \cancel{\pi} \times 6^2 \times 8$

$\Rightarrow 4n = 5^3 \times 6 \times 8 = 6{,}000$

$\therefore n = 1500$.

Thus, no. of lead shot = 1500.

22. Let the level of water in the tank rises by 35 cm in 't' hours.

Here, radius of the cylinder, $r = \frac{14}{2}$ cm $= 7$ cm $= 0.07$ cm.

Speed of the water = 5 km/hr = 5000 m/hr.

Length of the pipe, $h = 5000 \times t$ m.

Now, Vol. of water in the pipe = Vol. of water in the tank.

$\Rightarrow \pi r^2 h = 30 \times 22 \times \frac{35}{100}$

$\Rightarrow \frac{22}{7} \times 0.07 \times 0.07 \times 5000 \times t = 3\cancel{0} \times \cancel{22} \times \frac{\cancel{35}\,7}{\cancel{100}\,\cancel{20}}$

$\Rightarrow \frac{24.5}{7} t = \frac{21}{2}$.

$\Rightarrow t = \frac{21 \times 7}{24.5 \times 2} = \frac{147}{49} = 3$.

$\therefore$ Level of water in the tank rises by 35 cm in 3 hours.