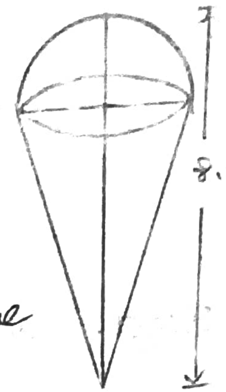
Exercise : 12.4.

1. Radius of hemisphere = 2.1 cm.

Height of the cone = 8.1 − 2.1 = 6 cm.



$\therefore$ Slant height, $l = \sqrt{2.1^2 + 6^2} = 6.35$ cm.

Thus, total surface area of the toy

= surface area of hemisphere + surface area of cone

$= 2\pi r^2 + \pi r l$.

$= \frac{22}{7}\left(2 \times 2.1^2 + 2.1 \times 6.35\right)$

$= \frac{22}{7} \times 22.155 \quad = \cancel{69}\ 69.63 \text{ cm}^2$.

And volume = volume of hemisphere + Volume of cone.

$= \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$.

$= \frac{\pi}{3}\left(2 \times 2.1^3 + 2.1^2 \times 6\right)$

$= \frac{22}{21} \times 44.98$

$= 47.12 \text{ cm}^3$.

2

Here, diameter $= 7$ cm.

$\therefore$ radius $= 3.5$ cm.

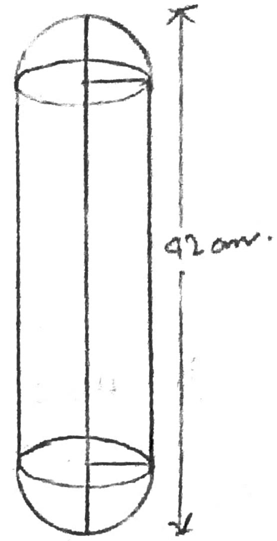
Then,

1) total curved surface area of the solid

$= 2\pi r h + 2 \times 2\pi r^2$.

$= 2\pi r\,(h + 2r)$

$= 2 \times \frac{22}{\cancel{7}} \times \cancel{3.5}^{0.5} \times 42 \quad = 924 \text{ cm}^2$



Now, volume of the solid

$= \pi r^2 h + 2 \times \frac{2}{3}\pi r^3$.

$= \pi r^2 \left(h + \frac{4}{3} r\right) \quad = \frac{22}{7} \times (3.5)^2 \times \left(35 + \frac{4}{3} \times 3.5\right)$

$= \frac{269.5}{7} \times \frac{1149}{3} \quad = 1527.16 \text{ cm}^3$.

3.

We have, $r = 2.1$ cm and $h = 8$.

Now, volume of icecream = vol. of cone + vol. of hemisphere.

$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \frac{1}{3}\pi r^2 (h + 2r) \qquad = \frac{1}{3} \times \frac{22}{7} \times 2.1^2 (8 + 2 \times 2.1).$$

$$= \frac{97.02}{21} \times 12.2 \qquad = 56.36 \text{ cm}^3.$$

4. Here,

Volume of cylinder = Vol. of an ice-cream × 16.

$$\Rightarrow \pi r^2 h = 16 \times (\text{Vol. of cone} + \text{Vol. of hemisphere})$$

$$\Rightarrow \cancel{\pi} \times 3^2 \times 14 = 16 \times \cancel{\pi}\left[\frac{1}{3} r_1^2 h + \frac{2}{3} r_1^3\right]$$

$$\Rightarrow 3^2 \times 14 = \frac{16}{3} \times r_1^2 (5r_1 + 2r_1) \quad [\because h = 5 \times \text{radius of bar}].$$

$$\Rightarrow 3^3 \times 14 = 16 r_1^2 \times 7 r_1$$

$$\Rightarrow r_1^3 = \frac{27 \times 14}{16 \times 7} = \left(\frac{3}{2}\right)^3.$$

$$\therefore r_1 = \frac{3}{2} \text{ cm}.$$

Then, volume of an ice cream = Vol. of cone + Vol of hemisphere.

$$= \frac{1}{3}\pi r_1^2 h + \frac{2}{3}\pi r_1^3 .$$

$$= \frac{\pi}{3}\left(r_1 \times 5r_1 + 2r_1^3\right) = \frac{\pi}{3} \times 7 \times r_1^3$$

$$= \frac{22 \times 7 \times 3 \times 3 \times 3}{7 \times 3 \times 2 \times 2 \times 2} = 24.75 \text{ cm}^3 .$$

5. Let x be the no. of icecream.

Then, by question,

volume of cylindrical vessel = $x \times$ vol. of an icecream.

$$\Rightarrow \pi \times 7^2 \times 30 = x\left(\frac{1}{3}\times\pi \times 3^2 \times 8 + \frac{2}{3} \times \pi \times 3^3\right)$$

$$\Rightarrow 49 \times 30 = 42x$$

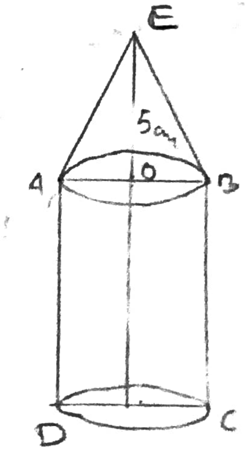
$$\therefore x = \frac{49 \times 30}{42} = 35 .$$

6. Here, $AD = BC = 4$ m.

$AB = DC = 24$ m

Radius $= \frac{1}{2} AB = 12$ m.

Now, vol. of the tent house



$= $ Volume of cylinder + volume of cone.

$= \pi r^2 h_1 + \frac{1}{3}\pi r^2 h_2$.

$= \pi r^2 \left(h_1 + \frac{1}{3}h_2\right) \qquad = \frac{22}{7} \times 12^2 \left(4 + \frac{5}{3}\right)$.

$= \frac{22}{7} \times 144 \times \frac{17}{3} \qquad = 2564.58\ m^3$.

And surface area of the tent house

$=$ surface area (cylinder + cone)

$= \pi r\,(2h + l)$.

$= \frac{22}{7} \times 12 \left(2\times4 + \sqrt{5^2+12^2}\right) \qquad = \frac{22 \times 12 \times 21}{7}$

$= 792\ m^2$.

$\therefore$ cost of canvas $=$ ₹ $792 \times 120 \qquad =$ ₹ $95{,}040$

7. Here, height of the tent $= 15m$.

diameter of the cylinder $= 32m$.

$\therefore$ radius $= 16m$.

height of cylinder $= 3m$.

$\therefore$ height of the cone $= 12m$.

Now, volume of the tent $= \pi r^2 h_1 + \frac{1}{3}\pi r^2 h_2$.

$$= \pi r^2 \left(h_1 + \frac{1}{3}h_2\right)$$

$$= \frac{22}{7} \times 16 \times 16 \left(32 + \frac{1}{\cancel{3}} \times \cancel{12}^{4}\right)$$

$$= \frac{22}{\cancel{7}} \times 16 \times 16 \times \frac{\cancel{4}3}{3} \qquad = 5632\ m^3$$

and area of canvas $= 2\pi rh + \pi rl. \quad = \pi r(2h + l)$.

$$= \frac{22}{7} \times 16 \left(2\times3 + \sqrt{12^2 + 16^2}\right)$$

$$= \frac{22 \times 16 \times 26}{7} \qquad = 1307.93\ m^2.$$

$\therefore$ cost of the canvas $=$ ₹ 110×1307.93 $=$ ₹ 143817.30.

8. Diameter of cylinder $= 12m$

$\therefore$ Radius $= 6m$.

height of cylinder $= 9m$.

Now, outer surface area of the hall

$$= 2\pi rh + \pi rl \quad = \pi r(2h + l).$$

$= \frac{22}{7} \times 6 \left(2 \times 4 + 6\sqrt{2}\right) \quad \left[\because \frac{r}{l} = \cos 45 \Rightarrow l = \frac{6}{1/\sqrt{2}} = 6\sqrt{2}\right]$

$= \frac{22}{7} \times 6 \left(8 + 8.46\right) \quad = \frac{22}{7} \times 6 \times 16.46$

$= 310.39 \ m^2$.

Then,

Vol. of the air inside the hall

$= \frac{1}{3}\pi r^2 h + \pi r^2 h'$ (h' = height of cylinder)

$= \pi r^2 \left(\frac{1}{3} h + h'\right) \quad = \frac{22}{7} \times 36 \left(\frac{1}{\cancel{3}} \times \cancel{6}^{2} + 4\right)$

$= 678.86 \ m^3$.

9\. Length of cylinder = 56 cm.

diameter of cylinder = 18 cm.

$\therefore$ radius = 9 cm.

Now, capacity of the geyser $= \pi r^2 h + \frac{4}{3}\pi r^3$

$= \pi r^2 \left(h + \frac{4}{3} r\right)$.

$= \frac{22}{7} \times 81 \left(56 + \frac{4}{\cancel{3}} \times \cancel{9}^{3}\right)$

$= \frac{22}{7} \times 81 \times 68 \text{ cm}^3.$

$= \frac{22 \times 81 \times 68}{7 \times 1000} \, l = 17.31 l.$

10. Here, height of cylinder, $h = 19cm$

radius of cylinder = radius of hemisphere, $r = 3cm$.

Now, the reqd. volume $= \pi r^2 h + \frac{2}{3}\pi r^3 = \pi r^2 (h + \frac{2}{3} r)$.

$= \frac{22}{7} \times 9 \left(19 + \frac{2}{\cancel{3}} \times \cancel{3}\right)$

$= \frac{22}{7} \times 9 \times 16 = 452.57 \text{ cm}^3.$

Then, internal surface area $= 2\pi rh + 2\pi r^2.$

$= 2\pi r (h + r)$

$= 2 \times \frac{22}{7} \times 3 \; (19 + 3).$

$= 2 \times \frac{22}{7} \times 3 \times 17.$

$= 320.57 \text{ cm}^2.$

11. Here, height of the cylinder, $h = 28$ cm.

radius $= 12$ cm.

We know, vol. of cone $= \frac{1}{3} \times$ vol. of cylinder

Now, Volume of the remaining solid

$= \frac{2}{3} \times \pi r^2 h$.

$= \frac{2}{\cancel{3}} \times \frac{22}{\cancel{7}} \times \overset{9}{\cancel{12}} \times 12 \times \overset{4}{\cancel{28}} = 8448 \text{ cm}^3$.

12. Here, radius of bigger cylinder, $r_1 = 14$ cm.

radius of smaller cylinder, $r_2 = 10$ cm.

height of cylinder $= 35$ cm.

Now, volume of the coke $= \pi r_1^2 h - \pi r_2^2 h$.

$= \pi h (r_1^2 - r_2^2) = \frac{22}{7} \times 35 (14^2 - 10^2)$

$= \frac{22}{\cancel{7}} \times \overset{5}{\cancel{35}} \times 96$.

$= 10560 \text{ cm}^3$.

13. Here, diameter of the cylindrical tub = 26 cm.

$\therefore$ radius, r_1 = 13 cm.

height of the cylinder, h_1 = 21 cm.

diameter of cone = 18 cm.

$\therefore$ radius, r_2 = 9 cm.

height of the cone ~~r_2~~ h_2 = 14 cm.

Now, quantity of water left in the cylindrical tub

= Vol. of cylinder − vol. of solid.

$= \pi r_1^2 h_1 - \left(\frac{1}{3}\pi r_2^2 h_2 + \frac{2}{3}\pi r_2^3\right)$

$= \pi \times \left\{13^2 \times 21 - \left(\frac{1}{3} \times 9^2 \times 14 + \frac{2}{3} \times 9^3\right)\right\}$

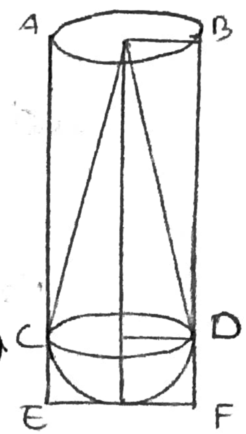
$= \frac{22}{7} \times (3549 - 864) = \frac{\frac{22}{7} \times 2685}{1000} = 8.44\, l.$

14. Let ABCD be the portion related to cone and CEFD be the portion related to hemisphere.

Then, height of cylinder ABCD = 6 cm = height of cone.

and height of cylinder CEFD = 2.8 cm

= radius of (cone & hemisphere)



Now reqd. space

$= $ (vol. of cylinder ABCD − vol. of cone) + (vol. of cylinder CEFD − vol. of hemisphere).

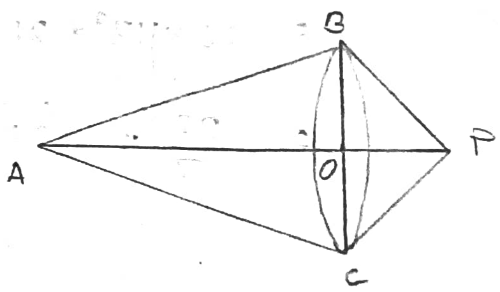
$= \frac{2}{3} \times$ vol. of ABCD $+ \pi \times 2.8 \times 2.8 - \frac{2}{3}\pi \times (2.8)^3$.

$= \frac{2}{3} \times \frac{22}{7} \times \overset{0.4}{\cancel{2.8}} \times 2.8 \times \overset{2}{\cancel{6}} + \left(\frac{22}{7} \times \overset{0.4}{\cancel{2.8}} \times 2.8 - \frac{2}{3} \times \frac{22}{7} \times 2.8^3\right)$.

$= 98.56 + 22.99$.

$= 121.56 \text{ cm}^3$,

15 Let AP be the hypotenuse. Taking OP = x cm



Then, $OA = (15 - x)$ cm

Now, $PB^2 = BO^2 + OP^2$

$\Rightarrow BO^2 = 81 - x^2$ — (1).

and $AB^2 = BO^2 + AO^2$.

$\Rightarrow BO^2 = 144 - (15 - x^2)^2$ — (11).

From, (1) and (11) we get

$\Rightarrow 81 - x^2 = 144 - 225 - x^2 + 30x$

$\Rightarrow 30x = 81 + 81$.

$\therefore x = \frac{162}{30} = 5.4$ cm.

hence, $OP = 5.4$ cm.

And $AO = 9.6$ cm and $BO = 7.2$ cm [using (1)]

Now, volume of the double cone.

$= \frac{1}{3} \times \frac{22}{7} \times 7.2^2 \times (9.6 + 5.4)$ cm^3

$= 814.63$ cm^3.

and surface area of the double cone

$= \pi r (12 + 9)$ cm^2

$= \frac{22}{7} \times 7.2 \times 21$ cm^2

$= 475.2$ cm^2.

16. Here,

Length of cuboid = 63 m
Breadth of cuboid = 28 m

Now, vol. of air inside the godown

$= $ vol. of cuboid + vol. of half-cylinder.

$= 5 \times 63 \times 28 + \frac{1}{2} \times \frac{22}{7} \times 14 \times 14 \times 63.$

$= 8820 + 19404 \quad = 28224\ m^3.$

and lateral surface area of the roof

$= \frac{1}{2} \times 2\pi r h \quad = \frac{1}{\cancel{2}} \times \cancel{2} \times \frac{22}{\cancel{7}} \times \overset{2}{\cancel{14}} \times 63.$

$= 2772\ m^2.$

$\therefore$ cost of roofing $=$ ₹ 2772×500 $=$ ₹ 13,86,000

17. As vol. of the solid = vol. of cylinder + vol. of the cone.

Now, Volume of solid $= \pi r^2(2h) + \frac{1}{3}\pi r^2 h.$

$= \frac{6\pi r^2 h + \pi r^2 h}{3} = \frac{7\pi r^2 h}{3}.$

$= \frac{7}{3}\pi r^2 h$ Proved.